Does Mathematics Instruction for Three- to Five-Year-Olds Really Make Sense?

Arthur J. Baroody

Over the past 25 years, researchers have accumulated a wealth of evidence that children between three and five years of age actively construct a variety of fundamentally important informal mathematical concepts and strategies from their everyday experiences. Indeed, this evidence indicates that they are predisposed, perhaps innately, to attend to numerical situations and problems.

It is important to note too that the mathematical ideas preschoolers construct are in some cases relatively abstract. Teachers of young children should be aware of the impressive informal mathematical strengths of children in the early years and recognize that it does make sense to involve them in a variety of mathematical experiences.

The purpose of this article is to review some of the recent research on young children's number and arithmetic concepts and skills. This research provides valuable insights into the extent of young children's mathematical learning—insights that can help us address the question of how best to provide mathematics experiences for young children.

What mathematics can three- to five-year-olds learn?

Over the course of the twentieth century, psychologists came to dramatically different conclusions about preschoolers' mathematical competence. Their focus shifted from trying to find what children *can't do* to trying to reveal what they *can do*.

Earlier views

Behaviorism. Behavioral theorists have shaped the conventional wisdom about mathematical teaching and children's learning to this day (Ginsburg, Klein, & Starkey 1998). Thorndike, for example, concluded that young children were so mathematically inept that "little is gained by [doing] arithmetic before grade 2, though there are many arithmetic facts that they can [memorize by rote] in grade 1" (1922, 198). According to association theorists, children had to be rewarded (bribed) to learn mathematics, understanding was not central to learning useful mathematical skills, and students had to be spoon-fed mathematics because they were uninformed and helpless. This view served as the rationale for the drill approach (Thorndike 1922) and, years later, shaped the doctrine of direct instruction (Bereiter & Engelmann 1966). The lecture-and-drill method remains to this day the most widely used way to teach children mathematics in the primary grades.

Piaget's constructivism. Jean Piaget's (1965) research offered a very different view of mathematical teaching and learning (Kamii 1985). In Piaget's constructivist view, young children have a natural curiosity. For example, they have an inherent desire to find patterns and resolve problems, the essence of mathematics.

For Piaget, the construction of mathematical understanding was the heart of real development in mathematics learning. For example, reflecting on the part-whole relations underlying addition, such as a whole is the sum of its parts and greater than any single part, advances mathematical thinking, while memorization of number facts by rote does not.

In Piaget's view, children actively construct their mathematical knowledge by interacting with their physical and social worlds. By listening to their parents, older siblings, and peers, for instance, children detect counting patterns, devise counting rules, and sometimes overapply these rules rather than passively absorb (imitate) the counting-word sequence they hear. A clear indication of this is rule-governed counting errors, such as when children say "fourteen, *fiveteen,* sixteen," or "twenty-eight, twenty-nine, *twenty-ten*"

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(Baroody with Coslick 1998). (It seems unlikely that any parents, siblings, or preschool teachers model and reward *fiveteen* or *twenty-ten*.)

Today abundant evidence (Ginsburg, Klein, & Starkey 1998) exists to support the views of Piaget. Interestingly, however, in Piaget's (1965) view, children so young are not capable of abstract concepts or logical thinking. Thus, he concluded, they are not capable of constructing a true concept of number or an understanding of arithmetic.

Recent developments

In the last 25 years, psychologists have revealed a more optimistic portrait of preschoolers' mathematical competence. Summarized below are findings regarding concepts fundamental to number sense and an understanding of school (formal) mathematics. By understanding what young children know about these foundational concepts and can do with them, teachers can incorporate developmentally appropriate activities to nurture children's mathematical development.

Informal concept of numerical equivalence. An ability to identify equivalent collections (e.g., recognizing ● ●, ★ ★, ❤ ❤, and ✣ ✣ all as pairs or 2; recognizing ★ ★ ★, ❤ ❤ ❤, and ✣ ✣ ✣ all as trios or 3, and so forth) is fundamental to understanding number. Research indicates that three-year-olds can already recognize equivalence between small collections of objects or pictured objects—that is, visually match collections or pictures of 1 to 4 items (e.g., Huttenlocher, Jordan, & Levine 1994). For example, they can identify ■ ■ ■ and ■ ■ ■ as "the same" and ■ ■ ■ and ■ ■ as "not the same." Moreover, four-year-olds, but not three-year-olds, can make auditory-visual matches such as equating the sound of three dings with the sight of three dots (Mix, Huttenlocher, & Levine 1996) and accurately compare sequential sets with static ones, such as three jumps by a puppet or three light flashes with three dots (Mix 1999).

What these results seem to indicate is that three-year-olds have already developed a nonverbal representation of number. Whether this representation consists of a mental picture, mental markers (analogous to tallies), or something else is not entirely clear. Nor is it clear whether this representation is an exact one or an estimate. In any case, the key finding is that three-year-olds already have a reasonably accurate way of representing and comparing small collections before they even learn to count them.

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Between three-and-a-half and four years of age, children's development of verbal and object-counting skills provides them a more powerful tool for representing and comparing numbers. In addition to allowing preschoolers to make comparisons of small sets, their counting-based representation enables them to compare collections larger than four items. Specifically, by counting and visually comparing small collections, children can recognize the *same number-name principle*: Two collections are equal if they share the same number name, despite differences in the physical appearance of the collection (Baroody with Coslick 1998). Because it is a general principle, young children can use it to compare any size collection that they can count.

Similarly by counting and visually comparing two unequal collections, preschoolers can further discover the *larger-number principle*: The later a number word appears in the counting sequence, the larger the collection it represents—for example, *five* represents a larger collection than *four* because it follows *four* in the counting sequence. Once children can automatically cite the number after another in the counting sequence—for example, the number after *four* is *five*—they can use the larger-number principle to mentally compare two numbers (e.g., Who is older, someone 9 or someone 8?—the nine-year-old because 9 comes after 8).

This relatively abstract number skill has many everyday applications and can be used for even huge numbers (1,000,129 is greater than 1,000,128 because, according to our counting rules, the former comes after the latter). Typically, children can name the number after another up to 10 and can use this knowledge and the larger-number principle to mentally compare any two numbers up to 5 before they enter kindergarten (four-and-a-half to five-and-a-half years of age). By the time they leave kindergarten (at five-and-a-half to six years old), children typically can compare any two numbers at least up to 10.

Informal addition and subtraction. An understanding of addition and subtraction is fundamental to success with school mathematics and everyday life. Recent research indicates that children start constructing an understanding of these arithmetic operations long before school. During the preschool years, they develop the ability to solve simple nonverbal addition or subtraction problems (Huttenlocher, Jordan, & Levine 1994).

Such problems involve showing a child a small collection (1 to 4 items), covering it, adding or subtracting an item or items, and then asking the child to indicate how many items there are now by counting out an appropriate number of disks. For one item plus another item (1 + 1), for instance, a correct response would involve counting out a set of two disks rather than, for instance, one disk or three disks.

In one study (Huttenlocher, Jordan, & Levine 1994), for example, most children who had recently turned three years old could correctly solve problems involving "1 + 1" or "2 – 1" (that is, they could imagine adding one object to another or could mentally subtract one object from a collection of two objects). By age four most children could mentally add and subtract small numbers of items such as "1 + 2," "2 + 1," "3 – 1," "3 – 2."

How do children so young manage these feats of simple addition and subtraction? They apparently can reason about their mental representations of numbers. For 2 + 1, for instance, they form a mental representation of the initial amount (before it is hidden from view), form a mental representation of the added amount (before it is hidden), and then can *imagine* the added amount added to the original amount to make the latter larger. In other words, they understand the most basic concept of addition—it is a transformation that makes a collection larger. Similarly, they understand the most basic concept of subtraction—it is a transformation that makes a collection smaller.

Later, but typically before they receive formal arithmetic instruction in school, children can solve simple addition and subtraction word problems (Huttenlocher, Jordan, & Levine 1994), including those involving numbers larger than 4. How do they manage this? Basically, children decipher the meaning of the story by relating it to their informal understanding of addition as a "make-larger" transformation or their informal understanding of subtraction as a "make-smaller" transformation (e.g., Carpenter, Hiebert, & Moser 1983; Baroody with Coslick 1998).

At least initially children use objects like blocks or fingers to model the type of transformation indicated by a word problem. Consider the following problem:

Rafella helped her mom decorate five cookies before lunch. After lunch she helped decorate three more cookies. How many cookies did Rafella help decorate altogether?

Young children might model this problem by counting out 5 blocks to represent the initial amount, counting out 3 more blocks to represent the added amount, and then counting all the blocks put out to determine the solution.

Research further reveals that children invent increasingly sophisticated counting strategies to determine sums and differences (Carpenter, Hiebert, & Moser 1983; Baroody with Coslick 1998). At some point, children abandon using objects and rely on verbal counting procedures. To solve the problem above, they might count up to the number representing the initial amount ("one, two, three, four, five") and continue the count three more times to represent the amount added ("six is one more, seven is two more, and eight is three more—eight cookies altogether"). One shortcut many children spontaneously invent is to start with the number representing the initial amount instead of counting from one: "Five; six is one more, seven is two more, and eight is three more—eight cookies altogether" (Baroody 1995).

As with making number comparisons, children's informal addition is initially relatively concrete in the sense that they are working nonverbally with real collections or mental representations of collections of 4 or less. Later, as they master and can apply their counting skills, they extend their ability to engage in informal arithmetic both in more abstract ways, such as word problems, and even later in symbolic expressions, such as 2 + 1 = ? , and with numbers greater than 4.

Part-whole relations. The construction of a *part-whole concept,* the understanding of how a whole is related to its parts, is an enormously important achievement. For example, it is considered the conceptual basis for understanding and solving missing-addend word problems, such as Problems A and B below, and missing-addend equations, such as ? + 3 = 5 and ? – 2 = 7.

- **Problem A.** Angie bought some candies. Her mother bought her three more candies. Now Angie has five candies. How many candies did Angie buy?

- **Problem B.** Blanca had some pennies. She lost two pennies while playing. Now she has seven pennies. How many pennies did Blanca have before she started to play?

Young children's inability to solve missing-addend word problems and equations has been taken as evidence that they lack understanding of the part-whole concept. Some have interpreted such evidence as support for Piaget's (1965) conjecture that the pace of cognitive development limits the mathematical concepts children can learn and have concluded that instruction on missing addends is too difficult to be introduced in the early primary grades (Kamii 1985).

Results of a study by Sophian and McCorgray (1994) suggest otherwise. These researchers gave four-, five-, and six-year-olds problems like Problems A and B. Problems were read to a child and acted out using a stuffed bear and pictures of items. Although five- and six-year-olds typically had great difficulty determining the exact answers to such problems, they gave answers that were at least in the right direction. For Problem A, for instance, they knew that the answer (a part) had to be less than 5 (the whole). For Problem B, they recognized that the answer (the whole) had to be larger than 7 (the larger of the two parts). These results suggest that five- and six-year-olds can reason about missing-addend situations and, thus, have a basic understanding of part-whole relations.

Equal partitioning. Equal partitioning is the process of subdividing a collection of items or a quantity, such as the surface area of a pizza, into equal-size parts. It is the conceptual basis for division, measurement, and fractions. Research (Davis & Pitkethly 1990) has shown that many children of kindergarten age can respond appropriately to fair-sharing situations or problems, such as the example below:

Three sisters, Martha, Marta, and Marsha, were given a plate of six cookies by their mom. If the three sisters shared the six cookies fairly, how many cookies would each sister get?

Some children solve this type of problem by using a divvying-up strategy; for example, children count out the 6 cookies to represent the amount, then deal out the cookies one at a time to each of 3 piles until all have been passed out, and then count the number in each pile to determine the solution. In effect, research suggests that even the operation of division can be introduced to children as early as kindergarten.

Informal fraction addition and subtraction. Perhaps most surprising of all is the research indicating that preschoolers can understand simple fraction addition and subtraction. Mix, Levine, and Huttenlocher (1999) presented to three-, four-, and five-year-olds nonverbal problems that involved, first, showing half of a circular sponge and then putting it behind a screen; next, showing half of another circular sponge and then also putting it behind the screen; and finally, presenting four

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choices—one-quarter of a sponge, one-half of a sponge, three-quarters of a sponge, and a whole sponge—and asking which was hidden behind the screen. The three-year-olds were correct only 25% of the time. They responded at a chance level—no better than could be expected by random guessing. The four- and five-year-olds, however, responded at an above-chance level. For instance, more than half were correct on problems involving "¼ + ½," "¼ + ¾," "½ – ¼," and "1 – ¼."

Conclusion

Preschoolers do have impressive informal mathematical strengths in a variety of areas as recent research indicates. Given this and their natural inclination for numerical reasoning, it makes sense to involve children in engaging, appropriate, and challenging mathematical activities. The key question is, How should preschoolers be taught mathematics?

How should preschoolers be taught mathematics?

In this section I first discuss a new approach for teaching mathematics and then delineate some key implications for early childhood mathematics instruction.

The investigative approach

Consistent with constructivist theory and its supporting evidence, the National Council of Teachers of Mathematics (NCTM 1989, 1991) has recommended shifting from a traditional instructional approach to an approach that better fosters the mathematical power of children. As will be evident below, this new approach is consistent with the teaching guidelines outlined in the revised edition of *Developmentally Appropriate Practice in Early Childhood Programs* (Bredekamp & Copple 1997).

What is mathematical power? Mathematical power has three components. The first is a positive disposition to learn and use mathematics. This includes the beliefs and confidence needed to tackle challenging problems. Teachers need to help children develop the belief that everyone is capable of understanding mathematics and solving mathematical problems.

The second element of mathematical power is understanding mathematics. This includes appreciating how school mathematics relates to everyday life, seeing the connections among mathematical concepts, and linking procedures to their conceptual rationale. To promote meaningful learning, then, teachers must help children (a) relate school-taught symbols and procedures to their informal, everyday experience; (b) consider how different ideas such as addition and subtraction are related (e.g., adding 1 can be undone by subtracting 1, and 3 – 2 = ? can be thought of as 2 + ? = 3); and (c) learn the whys as well as the hows of mathematics.

The third part of mathematical power is developing an ability to engage in the processes of mathematical inquiry. This includes making and testing conjectures, finding patterns in the world around us (inductive reasoning), problem solving, and logical (deductive) reasoning. An especially important but often overlooked process is communicating about mathematics. To promote the ability to engage in mathematical inquiry, teachers need to find challenging but developmentally appropriate problems and encourage children to discuss with others, including their peers, their suggestions for solving them and their solutions.

Why is mathematical power important? A positive disposition toward mathematics underlies the confidence and perseverance necessary to tackle challenging problems and lifelong learning of mathematics. Understanding greatly facilitates remembering and applying mathematics (Hatano 1988; Hiebert & Carpenter 1992; Rittle-Johnson & Alibali 1999). Meaningful learning requires less drill and practice than does learning by rote. Moreover, because children can apply what they understand, they can make connections and learn new material more easily on their own. Problem-solving and other inquiry skills are increasingly necessary in a progressively more complex world.

How can instruction foster mathematical power? A traditional instructional approach unfortunately robs children of mathematical power (Baroody with Coslick 1998). To better foster mathematical power, NCTM (1989, 1991) recommends that instruction be purposeful, meaningful, and inquiry based—what has been called the "investigative approach" (Baroody with Coslick 1998). In this approach instruction begins with a worthwhile task, one that is interesting, often complex, and *creates a real need* to learn or practice mathematics. Experiencing mathematics in context is not only more interesting to children but more meaningful—both of which make learning it more likely (Donaldson 1978; Hughes 1986).

In the investigative approach a teacher helps children build on what they already know to learn new concepts or procedures. By connecting new information or a problem to existing knowledge, children are far more likely to understand it. The instruction involves children in making conjectures, solving problems, inductive and deductive reasoning, and communicating their ideas, findings, or conclusions.

There is no better way to become proficient at these inquiry skills than to engage in real mathematical inquiries. Planning activities that are purposeful, meaningful, and inquiry based is at the heart of good early childhood teaching practice (Bredekamp & Copple 1997).

Implications for early childhood mathematics instruction

The investigative approach is particularly well suited for preschool children and their mathematics instruction. Below are some suggestions for making mathematics experiences for young children purposeful, meaningful, and inquiry based.

Purposeful instruction. There are a variety of ways to make mathematics instruction purposeful, such as using everyday situations, children's questions, games, and children's literature. Teachers can do so by finding and creating worthwhile tasks that create a real need for learning and practicing mathematics.

• *Everyday preschool activities provide numerous opportunities to learn or practice mathematics* (Kamii 1985; Baroody with Coslick 1998; Fromboluti & Rinck 1999). For instance, when preparing snack, table setters for each table can be asked to count the number of children in their group present that day to determine the number of place settings needed (counting a collection) and then put out for each child in the group a carton of milk, piece of fruit, paper plate, utensil, or whatever is needed (counting out a collection or one-to-one matching).

Note that such tasks might involve other skills such as addition (e.g., My group usually has five, but Clayton is sick today, so we have ____). Also note that teachers should not assign such tasks to children who are not developmentally ready (e.g., the atypical four-year-old who cannot verbally count to at least five or six) and should leave enough time to help those who are developmentally ready but who have not learned or mastered a needed skill.

• *Children's questions can provide invaluable teachable moments*. When Diane asks, "My birthday is next week, how old will I be? Will I be older than Barbara?" the teacher can respond by saying, "Class, Diane has some interesting questions with which she needs help. If she is three years old now, how can she figure out how old she'll be on her next birthday?" Note that the teacher asked the group how to solve the problem, she did not simply give the solution ("She will be four"). The teacher could then follow up by posing a problem involving both number-after and number-comparison skills: "If Barbara is five years old and Diane is four years old, how could we figure out who is older?"

Answering their own real questions can provide children with a powerful incentive to engage in mathematical inquiry and to explore and practice mathematical content. Furthermore, such conversations about mathematics provide teachers with a rich source of information about children's present and emerging understandings of number and arithmetic.

• *Games provide a natural, interesting, and structured way to explore or practice mathematics*. For children, play is a natural way of exploring their world and mastering skills for coping with it (Bruner, Jolly, & Sylva 1976). Playing math games can be an enjoyable way of raising interesting questions and practicing mathematical skills.

Teachers can choose or design math games to raise a particular issue or practice a particular skill (Kamii 1985; Baroody 1989; Baroody with Coslick 1998). For example, while playing Race Cars, Ari rolled a large balsa-wood die, and the side with 5 dots came up. He immediately recognized this as 5 (pattern recognition) and moved his car 5 spaces on the race track (counting out a collection of spaces). Bret then rolled the die and 5 dots came up again. He counted the dots (verbal counting and counting a collection) and, beginning with the space on which his car was resting, counted 5 spaces. "Hey," Bret complained, "I started on the same space as Ari, rolled a 5 like him, and now I'm behind him. How can that be?"

Through a discussion guided by the teacher, the players concluded that Bret counted as one the space he was on. The teacher asked Bret, "If you rolled a one, what would that mean?" Bret answered, "I could move one space." "Show me," asked the teacher. Again Bret started to count as one the space he was on but realized it would mean his car would not advance. To help Bret and others remember how to count out spaces correctly, the teacher recommended starting their counts with zero, using the whole-number sequence 0, 1, 2, 3, . . . , rather than the natural-number sequence 1, 2, 3, ... The game provided verbal- and object-counting practice, created a real need for discussing and correcting a common counting error, and brought about an opportunity for introducing the whole-number series.

• *Children's literature can provide another rich source of problems and content learning* (see Burns 1992; Thiessen & Matthias 1992; Whitin & Wilde 1992; Fromboluti & Rinck 1999). Consider, for example, *The Doorbell Rang* by Pat Hutchins (1986). The story begins with a mother presenting a plate of cookies to her two children and instructing them to share them fairly. Before reading on, a teacher could ask how this could be done and how many cookies each child would get. Pairs of children could be given chips, for instance, to model the situation, and each team could then share its strategy. Possibly at least one pair will suggest using an equal partitioning (divvy-up) strategy. As a follow-up activity, the children could role-play this story and others that involve mathematical concepts.

Meaningful instruction. Teachers should promote meaningful learning rather than learning by rote. They should promote and build on children's informal mathematical knowledge and help them see patterns and relations.

• *Foster and build on children's informal mathematical knowledge*. It is especially important to encourage preschoolers' use of verbal, object, and finger counting to represent, think about, and operate on numbers. Counting is a powerful tool for extending young children's nonverbal numerical and arithmetical competencies. Opportunities to learn and practice counting skill should be abundant, and children's use of counting solutions should be praised.

• *Focus on helping children see patterns and relations*. Instruction on mastering the verbal-counting sequence should concentrate on helping preschoolers discover counting patterns (e.g., the teen numbers are largely a repetition of the original sequence of numbers + the word *teen*: six + teen, seven + teen, . . .) and the exceptions to these patterns (e.g., "Although *fiveteen* is a good name for the number after fourteen, most people call it fifteen"). Playing an error-detection game in which a muppet or confused adult character tries to count and children help by pointing out errors such as ". . . nineteen, *tenteen*" is an enjoyable way to learn and practice counting rules and their exceptions.

Inquiry-based instruction. Teachers can foster positive dispositions toward mathematics by involving children in inquiry-based instruction. This includes promoting beliefs such as, "I can solve mathematical problems and do mathematics," which at heart are attempts to find patterns in order to solve problems. Inquiry-based instruction can also promote meaningful learning when, for instance, children discover a mathematical relation or listen to their peers' discoveries. Finally, it can help inquiry skills such as the ability to reason about and solve real or challenging problems.

Involving children in inquiry-based instruction means that teachers should encourage them to discover and do as much for themselves as possible. This does not mean that teachers should simply allow children to engage in free play all the time. Learning is more likely to occur if wiser adults or older children mediate younger children's experiences (Vygotsky 1968; Lave, Murtaugh, & de la Rocha 1984; Durkin et al. 1986; Saxe, Guberman, & Gearhart 1987; Leino 1990; Rogoff 1990; Blevins-Knabe & Musun-Miller 1996; Anderson 1997). Some ways teachers can mediate learning are noted below.

• *Regularly pose worthwhile tasks, offer thought-provoking and interesting questions or problems, and encourage children to pursue, answer, or solve them themselves.* A teacher asked Suzie, a kindergartner, what she thought was the largest number. Suzie quickly answered, "A million." The teacher then asked what the number after a million might be. The girl thought for a moment and responded, "A million and one." Asked what she supposed the next number might be, Suzie answered quickly, "A million and two—so there is no biggest number."

The teacher's questions prompted Suzie to reflect on her knowledge of numbers, apply her knowledge of counting rules to continue the counting sequence past a million, and then deduce from this experience that the counting sequence in theory could go on forever—that is, construct a concept of infinity.

• *In general, prompt children's reflection rather than provide feedback*. When children have difficulty arriving at a solution or arrive at an incorrect solution, provide hints, ask questions, or otherwise promote their thinking rather than simply give them the correct solution. For example, Kamie concluded that 5 and 2 more must be 6. Instead of telling the girl she was wrong and that the correct sum was 7, her teacher asked, "How much do you think 5 and 1 more is?" After Kamie concluded it was 6, she set about recalculating 5 and 2 more. Apparently, she realized that both 5 and 1 more and 5 and 2 more could not have the same answer. The teacher's question prompted her to reconsider her first answer.

• *Encourage peer-peer dialogue*. Other children can sometimes explain informal mathematical ideas or strategies to a child better than an adult can. Furthermore, sharing ideas with others can help children clarify their own thinking. Indeed, because peer-peer dialogues can often result in disagreements and disagreements can prompt children to reconsider their ideas, dialogue can be an invaluable way of advancing mathematical thinking.

Conclusion

Preschoolers are capable of mathematical thinking and knowledge that may be surprising to many adults. Teachers can support and build on this informal mathematical competence by engaging them in purposeful, meaningful, and inquiry-based instruction. Although using the investigative approach requires imagination, alertness, and patience by teachers, its reward can be increasing significantly the mathematical power of children.

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Suggested reading

For a more complete discussion of why the investigative approach makes sense and how to implement it, see Baroody with Coslick, 1998, in the "References" and the following resource:

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For ideas about teaching specific content in a purposeful, meaningful, and/or inquiry-based manner, see the aforementioned Baroody, 1989, and Kamii, 1985, in the "References" and the following resources:

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For further reading

Anderson, T.L. 1996. "They're trying to tell me something": A teacher's reflection on primary children's construction of mathematical knowledge. *Young Children* 51 (4): 34–42.

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Greenberg, P. 1993. Ideas that work with young children. How and why to teach all aspects of preschool and kindergarten math naturally, democratically, and effectively (for teachers who don't believe in academic programs, who do believe in educational excellence, and who find math boring to the max)—Part 1. *Young Children* 48 (4): 75–84; 1994—Part 2. *Young Children* 49 (2): 12–18, 88.

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